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# A Correlated- $k$ Based Ultra-Fast Radiative Transfer ( $k$ URT) Method

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## ABSTRACT

A new sensor-specific correlated- $k$  ( $c$ - $k$ ) ultra-fast radiative transfer (RT) formalism,  $k$ URT, has been designed for fast broad-bandpass scene simulations from UV-visible to LWIR wavelengths. A higher resolution RT code (1  $\text{cm}^{-1}$  MODTRAN) has been adapted to output 1  $\text{cm}^{-1}$  correlated- $k$  parameters for ozone, water, and the combined uniformly mixed species on a pressure-temperature grid, which are merged to form a compact  $c$ - $k$  set incorporating the sensor bandpass response function. The compact set is used to compute bandpass transmittance and radiance in near-real time. Scattering parameters (molecular Rayleigh, clouds and aerosols), blackbody and solar functions are cast as compact  $k$ -dependent source terms and used in the radiance computations. Preliminary transmittance results for 3-5 and 8-12 micron bandpasses and visible-MWIR sensors yield results within 2% of a 1  $\text{cm}^{-1}$  MODTRAN calculation with a two-orders-of-magnitude computational savings. Applications include near-earth broadband propagation and extinction calculations for target detection and recognition, mid-range tracking, and search and rescue operations from ground and low altitude aircraft.

**Keywords:**  $k$ URT, correlated- $k$ , instrument response function, sensor-specific bandpass  $k$ -distributions, radiance, transmittance, RT code, MODTRAN

## 1. INTRODUCTION

This paper presents preliminary results from  $k$ URT (Correlated- $k$ -distribution based Ultra-fast Radiative Transfer), a new fast and accurate radiative RT algorithm designed for scene generation and target acquisition by low flying aircrafts and land vehicles. Here, “fast” is defined as two or more orders of magnitude more rapid than 1  $\text{cm}^{-1}$  MODTRAN<sup>1</sup> calculations, and “accurate” is defined as an absolute transmission error no greater than 2% when compared to bandpass-averaged 1  $\text{cm}^{-1}$ -resolution MODTRAN calculations.

$k$ URT utilizes a sensor-specific bandpass  $k$ -distribution ( $k$  is the absorption coefficient) on a temperature and pressure grid to generate RT quantities (transmittance and radiance). Computation of the  $k$ -values is time-consuming, but is performed off-line prior to use. As an example of the time savings gained by this process, the 1- $\text{cm}^{-1}$  MODTRAN correlated- $k$  ( $c$ - $k$ ) procedure applied to the 3-5  $\mu\text{m}$  window (1334  $\text{cm}^{-1}$  wide) requires 22,678 individual  $k$ -distribution calculations. In contrast, the new algorithm requires only 17  $k$ -distribution calculations for the entire bandpass, in principle, a savings of three orders of magnitude in computational load.

$k$ URT is designed to perform rapid transmittance and radiance calculations for a very large number of lines-of-sight (LOS's) and a smaller number of model atmospheres (*e.g.*, as defined from Rawinsonde/radiosonde data) with a given sensor bandpass. A high-resolution  $kdb$  ( $k$ -database) is pre-calculated on a grid of pressure and temperature values using MODTRAN. For each new sensor response function, these data are convolved to form a compact  $kdb$  ( $ckdb$ ) for each atmospheric “species”. For each new model atmosphere, the species  $ckdb$   $k$ -values are (P, T)-interpolated and scaled. The results are used for the LOS RT computations by adjusting the layer  $k$ 's for path length.

The  $kdb$  files are independent of any particular atmosphere and will be supplied with  $k$ URT software. They will cover a large enough spectral range to encompass all sensor response functions of interest. Once these files are generated,  $k$ URT will not require MODTRAN.

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We have tested  $k$ URT with two relatively wide bandpasses, one in the 3-5  $\mu\text{m}$  region and another in the 8-12  $\mu\text{m}$  region, as well as with a series of substantially narrower bands in the visible-MWIR. A very wide bandpass means that *all* physical quantities, not just the absorption coefficients, may vary substantially in-band. The radiance algorithm requires modeling this spectral variation in quantities such as the blackbody function, solar irradiance function, and cloud, aerosol and molecular (Rayleigh) scattering properties. Among the molecular species,  $\text{H}_2\text{O}$  is a special case, as its self-broadening and self-continuum components scale quadratically with concentration while its remaining line and continuum components are linear with concentration. Finally, inter-species correlation, *e.g.*, the correlation between  $\text{CO}_2$  and  $\text{H}_2\text{O}$  transmittance in the 3-5  $\mu\text{m}$  region, dictates that the bandpass may need to be subdivided into two or more sub-regions (not necessarily contiguous) to achieve the desired accuracy.

## 2. ALGORITHM DESCRIPTION

### 2.1 Overview

$k$ URT starts by using a modified version of MODTRAN to output  $1 \text{ cm}^{-1}$   $k$ -distributions for  $\text{CO}_2$ ,  $\text{O}_3$ ,  $\text{H}_2\text{O}$ , and the remaining atmospheric species, which are lumped together as a single uniformly mixed (UMIX) species. A fixed  $\text{CO}_2$  concentration can be included within UMIK. These  $k$ -distributions are computed on a grid of temperature and pressure. For scenarios dominated by the lower atmosphere ( $< 18 \text{ km}$ ), the temperatures were chosen to be 305, 280, 255, 230, 205 and 180 K (6 values), and the pressures were selected to be 1.0, 0.82, 0.64, 0.46, 0.28 and 0.1 atm (6 values). A finer (P, T)-grid will be needed to accurately model  $\text{O}_3$ . The result is 36 high-resolution  $k$ -distributions per species for each  $1 \text{ cm}^{-1}$  bin of the bandpass, which can be used for any sensor response function within the covered wavenumber range. For example, a database appropriate for the 3-5 and 8-12  $\mu\text{m}$  regions would cover the 3-12  $\mu\text{m}$  region ( $2500 \text{ cm}^{-1}$  wide), and each species kdb file would contain  $2500 \times 36 \times 17 = 1,530,000$   $k$ -values assuming 17  $k$ -values per  $1 \text{ cm}^{-1}$  bin.

Each high-resolution kdb file is convolved with sensor response functions to create compact kdb (ckdb) files as described below. Since this process creates just 17 values for the entire bandpass, each ckdb file contains only  $36 \times 17$   $k$ -values. Both the kdb and ckdb files are created off-line; only the latter is used at run time for the RT calculations.

Each species ckdb file may be used to compute LOS transmittance for that species. Under the assumption of spectral uncorrelation amongst the species, the total in-band transmittance is the product of the species transmittances. We have found that in order for this assumption to hold, a broad bandpass band may need to be subdivided into a small number (typically 2-3) of sub-bands.

The geometry module computes layer species column densities and average (P, T) for each LOS. The instrument-response  $k$ -values are obtained by (P, T)-interpolation of data in the ckdb files and then by appropriate scaling by species concentration and layer path length. The  $j^{\text{th}}$  ( $j=1, 17$ )  $k$ -values of each layer of the LOS are summed up and finally the transmittance is computed by the formula:

$$T = \sum_{j=1}^{j_{\max}=17} \Delta g_j \exp \left( - \sum_{\text{layer}} k_{\text{layer},j} u_{\text{layer}} \right) \quad (1)$$

where  $\Delta g_j$  is the weighting factor associated with the  $j$ -index,  $u$  is the absorber amount, and the product  $ku$  denotes the unitless quantity of optical depth. Obtaining  $k_j$  for the whole LOS by summing the layer  $k_j$  contributions is based on the fundamental assumption of correlation across layers. This assumption, which is accurate for typical atmospheric temperatures and pressures, is basic to  $c$ - $k$  applications.

For fast radiance calculations, it is necessary to develop a combined  $k$ -distribution for all atmospheric species, as well as  $k$ -dependent parameters that enter into the calculation of source functions. The procedures for doing this are described in Sections 2.4 and 2.5.

### 2.2 MODTRAN $c$ - $k$ Implementation

The  $c$ - $k$  formalism<sup>2,3</sup> was incorporated<sup>4</sup> in MODTRAN to recast band model transmittance into a monochromatic Beer's Law form for more accurate interfacing with scattering codes such as DISORT<sup>5</sup>. According to the  $c$ - $k$  formalism, the integral over frequency is transformed from frequency space ( $\nu$ ) into cumulative probability space ( $g$ ). In this

transformed space, the function  $g(k)$  is defined as the probability that an absorption coefficient ( $k$ ) in the defined frequency range will be less than or equal to  $k$ . The expression for transmittance then becomes

$$T = \frac{1}{\Delta\nu} \int \exp(-k_\nu u) \cdot d\nu = \int \exp(-k(g) \cdot u) \cdot dg \approx \sum_{j=1}^{j_{\max}=N} \exp(-\bar{k}(g_j) \cdot u) \cdot \Delta g_j \quad (2)$$

where  $\sum_j(\Delta g_j)=1$  and  $\bar{k}(g_j)$  is the average value of  $k(g)$  in the  $\Delta g_j$  interval.

The  $k$ -distributions in MODTRAN are generated from band model parameters at the native resolutions of  $\Delta\nu = 0.1, 1, 5$  or  $15 \text{ cm}^{-1}$ ; the  $k$ URT formalism uses the  $1\text{-cm}^{-1}$  resolution. In MODTRAN, a database of  $k(g)$  as a function of Doppler and Lorentz line-width parameters is pre-computed on a discrete grid of  $\Delta g_j$ , using a Monte-Carlo method for randomly distributing  $n$  number of lines within  $\Delta\nu$ ;  $n$ , which ranges from 1 to 90 per  $1 \text{ cm}^{-1}$ , encompasses the range of the band model line-density parameter values. The  $\Delta g_j$  were initially chosen so that the  $k(g)$  distribution with the largest dynamic range in  $k$  would have each decade of  $k$  covered by three discrete  $k$ -values. This amounted to 33 ( $k, \Delta g$ )-pairs per spectral bin and species. This number was later reduced to 17 pairs with minimal loss of accuracy.

During a MODTRAN  $c$ - $k$  execution, a set of layer  $k_j$  (for each spectral bin) are computed based on species-averaged line-width, line-strength and line-density parameters using the values stored in precomputed  $c$ - $k$  and band model databases. At each layer the individual species parameters are combined to generate  $k$ -distributions for a single effective pseudo gas species. The layer  $k$ -values are adjusted via a scaling factor to make the layer transmittances match band model theory.

### 2.3 $k$ URT Transmittance Calculation – Single Species

We now derive an expression for total bandpass transmission for a single species beginning with the transmittance for the  $i^{\text{th}}$  single frequency bin.

At the outset, the notation needs to be clarified: Index  $i$  and associated variables refer to individual  $1 \text{ cm}^{-1}$  bins of the bandpass. Index  $j$  and associated variables refer to  $c$ - $k$  terms within an arbitrary  $1 \text{ cm}^{-1}$  bin. Index  $l$  and associated variables refer to all pairs of ( $i, j$ ) where  $i$  runs over each bin and  $j$  runs over  $c$ - $k$  terms in each bin; *i.e.*,  $l$  runs over  $i_{\max} \times j_{\max}$  number of  $c$ - $k$  terms encompassing the entire bandpass. In what follows, we introduce a new index  $m$  which runs over down-sampled  $c$ - $k$  terms for the entire bandpass. For ease of coding and compatibility with MODTRAN the down-sampled  $\Delta g_m$  set is chosen to be identical to the  $\Delta g_j$  set, with both having 17 members.

The transmittance for the  $i^{\text{th}}$  single frequency bin is given by

$$T_i = \sum_{j=1}^{j_{\max}} \exp(-\bar{k}_{i,j} u) \cdot \Delta g_j \quad (3)$$

where  $j$  is the index of the ( $k, \Delta g$ )-pair within the bin. Including an instrument response (filter) function, *normalized* via

$$\sum_i f_i \equiv \sum_i \left( \int_{(i-1/2)\Delta\nu}^{(i+1/2)\Delta\nu} f(\nu) d\nu \right) = 1 \quad (4)$$

leads to the bandpass transmittance given by

$$T_f = \sum_i f_i T_i = \sum_i f_i \cdot \sum_{j=1}^{j_{\max}} \Delta g_j \exp(\bar{k}_{i,j} u) \quad (5)$$

We define a cumulative instrument-weighted probability  $g$  for the entire bandpass such that

$$\Delta g_\ell = f_i \cdot \Delta g_j \quad (6)$$

where  $\ell$  is the index of the ( $k, \Delta g$ )-pair within the bandpass (*i.e.*,  $\ell$  runs from 1 to  $i_{\max} \times j_{\max}$ , and it represents the total number of  $c$ - $k$  terms). The bandpass probabilities sum to unity:

$$\sum_{\ell=1}^{i_{\max} \cdot j_{\max}} \Delta g_{\ell} = 1 \quad (7)$$

The equation for in-band transmittance now becomes:

$$T_f = \sum_{\ell=1}^{i_{\max} \cdot j_{\max}} \Delta g_{\ell} \cdot \exp(\overline{k_{\ell} u}) \quad (8)$$

where the sum over  $\ell$  encompasses all combinations of  $(i, j)$  pairs. The next step is to down-sample the  $k_{\ell}$ -values to a manageable few for the entire bandpass. As mentioned above, for ease of coding and compatibility with MODTRAN, the down-sampled set  $\Delta g_m$  set is chosen to be identical to the 17-member  $\Delta g_j$  set.

The process of down-sampling starts with creating a new cumulative probability,  $g_l(k)$  for all  $k_{\ell}$ -values of the entire bandpass. Sorting the  $k_{\ell}$ -values in increasing order, assigning to each  $k_l$  the weight  $\Delta g_l$  and defining a new function

$$g_l(k) = \sum_{l'=1}^l \Delta g_{l'}, \quad \text{using the sorted set } g_l(k) \text{ gives the new cumulative probability function } g_l(k). \text{ The cumulative probability}$$

function is the inverse of the newly sorted  $k$ - $g$  curve. Whether the  $k_l$  are sorted or not, Eq. (8) remains valid. Assuming that Eq. (8) contains the  $k_{\ell}$ -values in increasing order, down-sampling leads to the following expression for  $T_f$ :

$$T_f = \sum_{m=1}^{m_{\max}} \Delta g_m \cdot \exp(\overline{k_m u}) \quad (9)$$

where  $\overline{k_m}$  is defined using the newly created  $g_l(k)$  and using the  $\Delta g_m$  discretization of the cumulative probability:

$$\overline{k_m} = \frac{\sum_{l=1}^{g_{m-1} \leq g_l < g_m} (\Delta g_l \cdot k_l)}{\sum_{l=1}^{g_{m-1} \leq g_l < g_m} \Delta g_l}, \quad g_l = \sum_{l'=1}^l \Delta g_{l'}, \quad g_m = \sum_{m'=1}^m \Delta g_{m'} \quad (10)$$

#### 2.4 $k$ URT Radiance Algorithm – Single Species

In the simplest approximation, optical quantities  $\mathcal{G}(\nu)$  other than spectral line extinction coefficients are assumed to have little variation within the bandpass, so that their effective values may be represented by the bandpass average,  $\overline{\mathcal{G}}$ , weighted by the filter function:

$$\overline{\mathcal{G}} = \frac{\int f(\nu) \cdot \mathcal{G}(\nu) \cdot d\nu}{\int f(\nu) \cdot d\nu} \quad (11)$$

Using this procedure for including thermal quantities (the blackbody function  $B_{\nu}$ ) and the solar irradiance function ( $I_{\nu}$ ) in the RT calculations provides good agreement with higher-resolution algorithms in the visible and near infrared for lower atmospheric temperatures. However, poor agreement is obtained with wide bandpasses in the mid-wave infrared (MWIR), where the blackbody function can vary by an order of magnitude or more and is a significant contributor to overall radiance. Here, it is important to incorporate the spectral correlation between the blackbody function and the molecular absorption coefficients. A method for doing this is described below. This procedure can be generalized for the solar function, scattering cross-sections, or any other frequency-dependent quantity needed for RT calculations.

The bandpass-integrated thermal radiance *at the front* of a layer is given by

$$R = \frac{1}{\Delta \nu} \int B_{\nu} \cdot (1 - T_{\nu}) \cdot f(\nu) \cdot d\nu = \sum_i f_i \cdot \sum_{j=1}^{j_{\max}} \overline{B}_i \cdot (1 - \overline{T}_{i,j}) \cdot \Delta g_j \quad (12)$$

where  $\overline{B}_i$  is the blackbody function averaged within the  $i^{\text{th}}$  bin. We can then rewrite Eq. (12), using the notation of the previous section, as follows:

$$R = \sum_{l=1}^{i_{\max} \cdot j_{\max}} \overline{B}_l \cdot (1 - \overline{T}_l) \cdot \Delta g_l, \quad \overline{T}_l = \exp(-k_l u) \quad (13)$$

Note that  $\overline{B}_i$  is independent of subscript  $j$  and dependent only on subscript  $i$ . In the above equation, we can assume that  $k_i$  are sorted and we have a new cumulative probability function  $g_i(k)$ . Down-sampling the new probability function, we can rewrite

$$R = \sum_m \overline{B}_m \cdot (1 - \overline{T}_m) \cdot \Delta g_m \quad (14)$$

where  $T_m = \exp(-\overline{k}_m u)$ ,  $k_m$  is as previously defined, and  $\overline{B}_m$  is defined as  $\Delta g_l$ -weighted average blackbody function:

$$\overline{B}_m = \frac{\sum_{l=1}^{g_{m-1} \leq g_l < g_m} (\Delta g_l \cdot \overline{B}_l)}{\sum_{l=1}^{g_{m-1} \leq g_l < g_m} \Delta g_l} \quad (15)$$

The resulting layer radiance values  $R$  from Eq. (14) are combined with their corresponding path transmittances to form the total path radiance, as is done in MODTRAN.

## 2.5 Combining ( $k$ , $g$ )-Values from Multiple Species

As noted earlier, total transmittance may be rapidly calculated from the product of transmittances derived from the species  $c$ - $k$  databases. However, many applications, notably scattering computations, require a single ckdb set, for each layer, that includes all species. One may derive such a ckdb from a convolution of the species ckdb's, which obeys the transmittance product (spectral uncorrelation) assumption.<sup>2</sup> The convolution gives the joint probability distribution of the sum of any two absorption coefficients:

$$k_A \otimes k_B \equiv \sum_m \sum_n (k_A(m) + k_B(n)) \cdot \delta g_m \cdot \delta g_n \quad (16)$$

This leads to  $N^2$  ( $k$ ,  $\Delta g$ )-pairs. To maintain the small number of  $k$ -distribution computations at runtime, the convolved absorption coefficients are re-sorted in ascending order and then down-sampled. This step must be performed at run time, as the absorption coefficients are a function of particle density and path length, which are not known until an atmosphere and LOS is defined. Indeed, this is done for each layer to compute transmittances (for radiance computations). For maximum speed advantage, the convolution and resampling are performed sequentially for each species; *i.e.*, the procedure is done on the first two species, the down-sampled result is again convolved and down-sampled with the third species, and so on. We have verified that the results are essentially identical to those from the transmittance product method.

A complication of using a combined-species ckdb is the lack of a general prescription for assigning  $k$ -dependent optical quantities  $\mathcal{A}(\nu)$  to the resulting ( $k$ ,  $\Delta g$ )-pairs. However, for scattering cross-sections and associated quantities (*i.e.* the phase function and single-scattering albedo) a straightforward solution exists, since the scattering originates almost entirely from a single species, UMIX. The  $k$ -dependence of these quantities is defined by keeping track of how the UMIX ckdb values enter into the combined-species ckdb values. To save computation time, this step is pre-calculated using the ground level molecular densities.

## 3. INITIAL RESULTS

### 3.1 3-5 $\mu\text{m}$ (2000-3334 $\text{cm}^{-1}$ ) Single Species Transmittance

Single-species (UMIX including  $\text{CO}_2$ ,  $\text{O}_3$  and  $\text{H}_2\text{O}$ ) transmittance calculations for the 3-5  $\mu\text{m}$  bandpass were computed for vertical and 45 degree off-nadir paths between 100 km and the ground. For the 1976 US Standard Atmosphere, the maximum absolute error is 0.0047 in  $\text{H}_2\text{O}$  transmittance (for 45° slant path), and maximum percent relative error is less than 0.7 % (the  $k$ URT and MODTRAN, Version 4, values were 0.6932 and 0.6979, respectively).

### 3.2 3-5 $\mu\text{m}$ Total Transmittance Results

Under the uncorrelation assumption, the total bandpass transmittance is given by:

$$T_f = T_f^{\text{H}_2\text{O}} T_f^{\text{UMIX}} T_f^{\text{O}_3} \quad (17)$$

The uncorrelation assumption has been found to hold well for bandpasses or spectral bins that are narrow compared to vibrational band widths. Examples are the spectral bins used in MODTRAN, which range in width from 0.1  $\text{cm}^{-1}$  to 15  $\text{cm}^{-1}$ . However, for  $k$ URT the uncorrelation assumption has been proposed to apply to much wider bandpasses, such as one covering the entire 3-5  $\mu\text{m}$  region, a range of over 1000  $\text{cm}^{-1}$ . Using the above formula, we computed the  $k$ URT

transmittance for the 3-5  $\mu\text{m}$  bandpass and compared the results to MODTRAN. The results, not too surprisingly, were not within desired accuracy;  $k$ URT transmittances were higher by about 0.03.

The reason for this discrepancy lies in the spectral variation of species transmittances shown in Figure 1, where one finds strong anti-correlation between water and the remaining species, notably  $\text{CO}_2$ . We proposed and tested a remedy to this problem, which is to subdivide the 3-5  $\mu\text{m}$  interval into two or more spectral intervals having different transmittance relationships. Very good results were obtained with MODTRAN-calculated transmittances using just two disjoint sub-bands, one corresponding to strong water absorption and the other corresponding to weak water absorption, as shown in Figure 1. For the  $k$ URT implementation we chose a slightly different and simpler division of the 3-5  $\mu\text{m}$  band. The first region was 2000-2900  $\text{cm}^{-1}$  (3.4-5.0  $\mu\text{m}$ ), where there is strong  $\text{CO}_2$  absorption and generally weak water absorption, and the second was 2900-3334  $\text{cm}^{-1}$  (3.0-3.4  $\mu\text{m}$ ), where there is weak  $\text{CO}_2$  absorption and generally strong water absorption.

The total bandpass transmittances for both the vertical and slant paths are shown in Table 1 and Figure 2. The total transmittance is calculated by weighting, according to their spectral width, the sub-bandpass transmittances of the two spectral segments (weight = 0.6747 for 2000-2900  $\text{cm}^{-1}$  and 0.3253 for 2900-3334  $\text{cm}^{-1}$ ). For brevity, we have not reported here the transmittances for the two sub-bands individually. Interestingly, we have observed that the error in total transmittance for the tropical vertical path is smaller than that for the 2000-2900  $\text{cm}^{-1}$  sub-band. This is not surprising, as the sub-band transmittances are unequally weighted and their errors are not all in the same direction or of the same size.

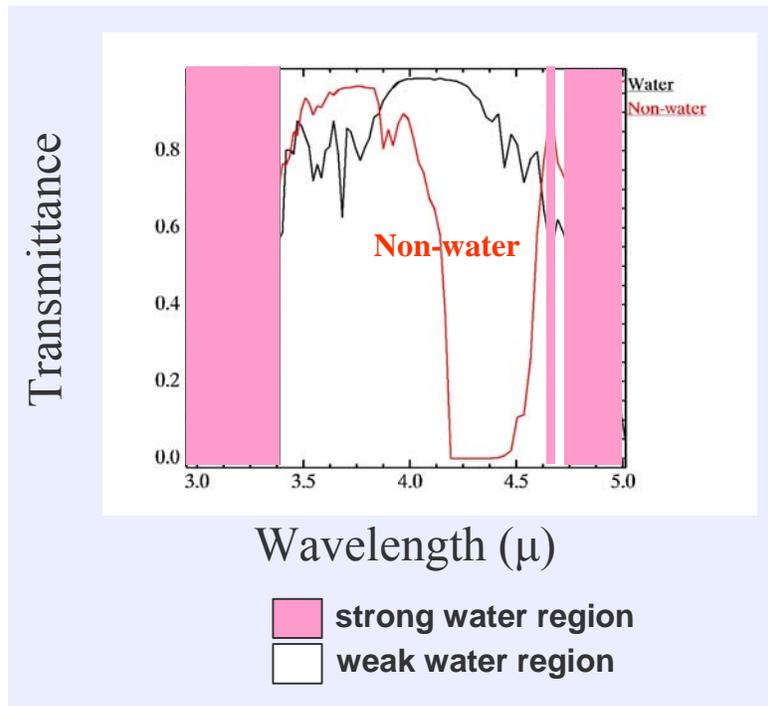


Figure 1: Spectral variation (transmittance versus wavelength in microns) of MODTRAN water and non-water species transmittances for a vertical path from space to ground using the Tropical atmosphere.

Table 1: Total 3-5  $\mu\text{m}$  (2000-3334  $\text{cm}^{-1}$ ) bandpass transmittances for the **vertical** and **slant path** to ground.

Path	Tropical		Mid-Lat Summer		Mid-Lat Winter		Sub-Arctic Summer		Sub-Arctic Winter		US Standard	
	$k$ URT	MOD	$k$	M	$k$	M	$k$	M	$k$	M	$k$	M
Vertical	0.4031	0.3909	0.4381	0.4284	0.5539	0.5467	0.4764	0.4679	0.6059	0.6012	0.5105	0.5020
Slant Path	0.3490	0.3351	0.3826	0.3714	0.4998	0.4910	0.4204	0.4104	0.5553	0.5494	0.4546	0.4449

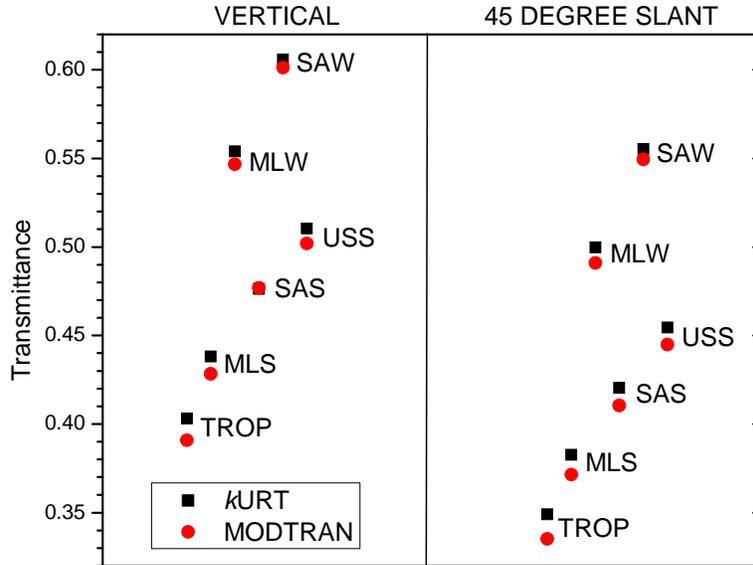


Figure 2: The 3-5  $\mu\text{m}$  band transmittances using  $k\text{URT}$  and MODTRAN for the six MODTRAN model atmospheres for two viewing geometries. The six atmospheres are Sub-Arctic Winter (SAW), Mid-Latitude Winter (MLW), 1976 U.S. Standard (USS), Sub-Arctic Summer (SAS), Mid-Latitude Summer (MLS) and Tropical (TROP). The two viewing geometries are a vertical path from space to ground and a  $45^\circ$  off nadir path from 100 km to ground.

### 3.3 8-12 $\mu\text{m}$ ( $830\text{-}1250\text{ cm}^{-1}$ ) $\text{H}_2\text{O}$ Transmittance Results

The 8-12  $\mu\text{m}$  region presents a special challenge for  $k\text{URT}$  because of the strong influence of the water vapor self-continuum, which scales quadratically with concentration. Treating all of the water vapor spectral components (Figure 3) as a single species requires referencing the self-continuum to a specific water vapor profile in the kdb. Some error in the self-continuum amount is therefore introduced when this kdb is used with a different profile. The problem may be avoided by treating the self-continuum and the remaining components (line+foreign-continuum) with two separate  $k$ -distributions, each with their own concentration scaling. The two  $k$ -distributions may be combined assuming either spectral uncorrelation (*i.e.*, treating them as two independent species) or full correlation. There may be error with either choice, as the actual degree of correlation is in between these limits.

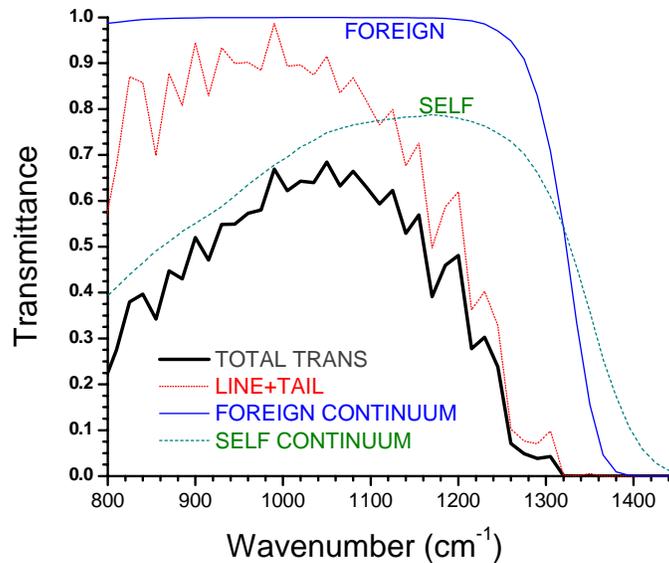


Figure 3: Water transmittance, vertical, space-to-ground, components.

Figure 4 shows the vertical ground-to-space transmittance for the two separate water vapor components and for the total. For each component the  $k$ URT transmittance compares extremely well against MODTRAN. The product of the  $k$ URT line+foreign (linear-in-concentration scaling) and self-continuum (quadratic-in-concentration scaling) transmittance values is in close agreement with total H<sub>2</sub>O transmittance calculated by MODTRAN; the largest error is 0.0051 for the Tropical atmosphere. Thus, for this bandpass the assumption of spectral uncorrelation between the self-continuum and the other water vapor components holds well. The alternative, full correlation assumption yields errors that are several times larger. Those errors may however be mitigated by subdividing the spectral region into several sub-bands.

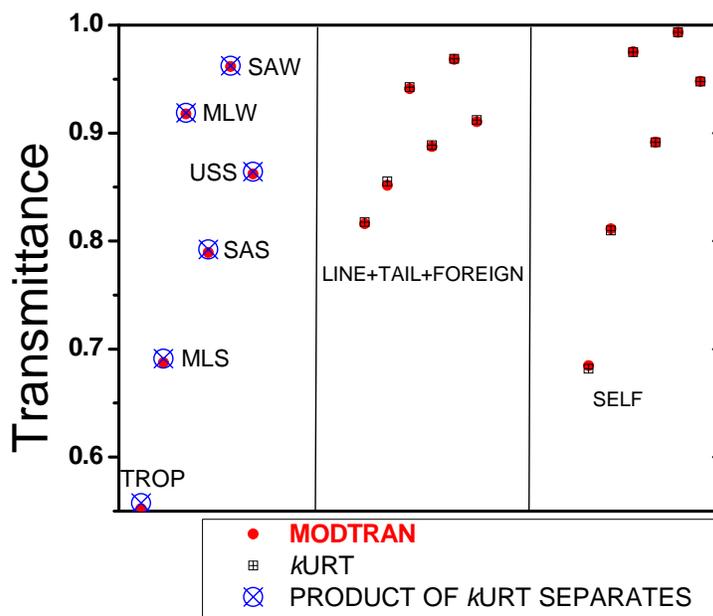


Figure 4: Modeling water transmittance, vertical, space-to-ground, by treating the totality of line+tail+foreign components separately from the self-continuum component. The left panel shows the total MODTRAN transmittances and  $k$ URT transmittances (product of the two  $k$ URT components). The middle panel shows line+tail+foreign components, and the third panel on the right shows the self-continuum component. From bottom to top, the six MODTRAN atmosphere models are: Tropical, Mid-Latitude Summer, Sub-Arctic Summer, US Standard, Mid-Latitude Winter and Sub-Arctic Winter. The order also corresponds to decreasing amount of water column density; the Tropical atmosphere has the most water, and the Sub-Arctic Winter has the least amount of water.

### 3.4 Visible-MWIR Radiance

Visible-NIR radiance calculations are in progress using a modified version of MODTRAN that incorporates the ckdb files and  $k$ URT elements to compute all RT quantities. In the initial code, bandpass-averages are used for the blackbody, solar and scattering terms. Calculations to date have been performed for visible through MWIR bandpass filters under daytime conditions with model atmospheres having a wide range of water vapor and visibility (aerosol amount). Multiple scattering calculations were performed using the DISORT<sup>5</sup> option. Maximum error in radiance was found to be about 5%, with typical errors of 2.5% or less. Transmittance errors were consistently less than 2%. As expected, computational savings relative to 1 cm<sup>-1</sup> MODTRAN were proportional to the number of  $k$ -values employed, and hence to the width of the bandpass. Detailed results will be presented in a future paper.

## 4. CONCLUSIONS

An instrument-response-function-specific correlated- $k$  approach, dubbed  $k$ URT, has been developed to rapidly compute bandpass LOS transmittances and radiances from the visible to the IR. In its full implementation,  $k$ URT will be a complete RT code that includes scattering due to aerosols and clouds. The goal is a hundred- to thousand-fold speed-up with 2% accuracy in RT quantities compared to a 1 cm<sup>-1</sup> MODTRAN run. Preliminary results on several wide bandpasses and narrower visible-MWIR instrument response functions indicate that both accuracy and speed goals are within reach.

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